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THE MATHEMATICS TEACHER

Devoted to the interests of mathematics in Elementary and Secondary Schools

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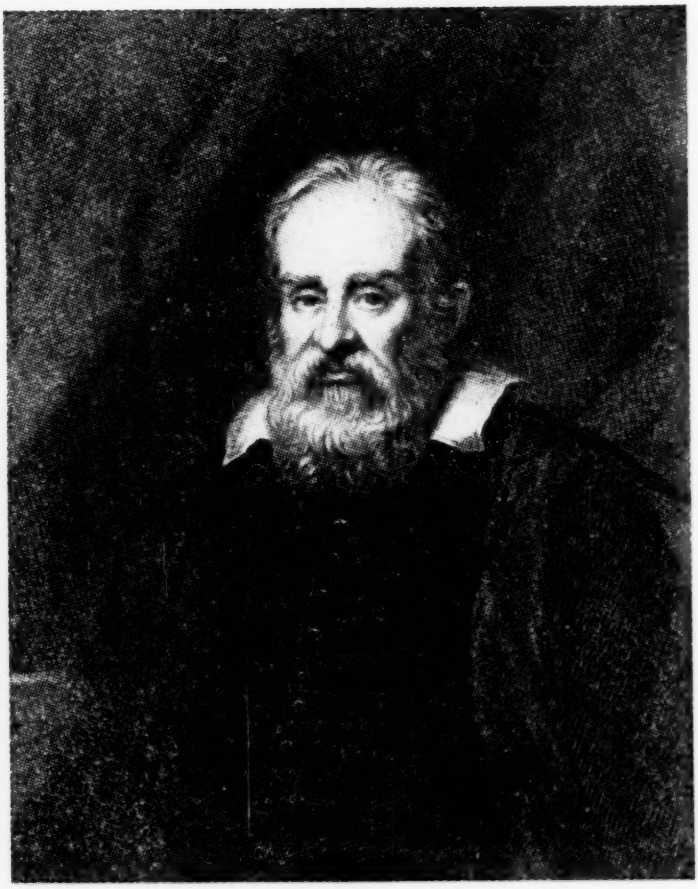

Galileo

THE MATHEMATICS TEACHER

Volume XXIV Number 2

Edited by William David Reeve

A Testing Program in Elementary Algebra and Its Evaluation

By P. L. WELTON

Benjamin Franklin High School, Rochester, N.Y.

INTRODUCTION—Results obtained by pupils on various tests and examinations in algebra have been published from time to time. The use of such results and the application of remedial measures to insure better teaching and a higher degree of success on the part of more pupils was outlined, for example, in Mr. J. B. Orleans' article in the March 1930 issue of *The Mathematics Teacher*.

It is not my intention to duplicate data of this sort. Rather it is my purpose to describe the type of testing in algebra which is in use in one high school in Rochester, and to submit some actual results obtained in a recent test of this kind.

Method of Preparing Tests—For several years, a series of departmental tests has been given at East High School on the principal units of each term's work in mathematics in Grades 9, 10, and 11. These tests are made out by a "subject-chairman" of each term's work. The individual teachers, after giving the tests, report to the subject-chairman the frequency distribution, the median, and the question analysis for their respective classes on each test. The results of each test are summarized by the subject-chairman, and the departmental results as well as the individual reports are returned

to the teachers. It is intended that these results shall be sent to the subject-chairman and the summarized report returned to the individual teacher as quickly as possible after each test, so that remedial work can be done for types of exercises that have proved difficult.

*Testing Program of 9-B Algebra Classes**

The testing program in 9-B Algebra for the January 1930-June 1930 semester is described in the following pages, together with the results obtained on the final examination.**

Three cumulative tests were given in the ten 9-B Algebra classes during the term. These tests were given at approximately five-week intervals. The *first* cumulative test covered the use of letters in algebra, the use of formulas, and the making of formulas; the *second* test covered the work on equations and simple problems; the *third* covered the work on the graph, signed numbers, some of the fundamental operations, and a short review of equations and problems. The results obtained on these tests will not be summarized here. They are on file at East High School and are open to inspection. The tests were given during an ordinary class period of forty minutes. They involved a rather searching examination of all the major skills and processes under consideration. Because of the limited time allotment given to these tests and because of the element of recency involved in giving them, no detailed comparison is given here of the results obtained on these cumulative tests with those obtained on the final examination. However, *all the results obtained during the semester were in striking harmony with the results obtained on the final examination.* As might have been expected, the medians on the cumulative tests were slightly higher than on the final examination. Naturally, the content of the final examination was influenced

* 9-B indicates first semester's work.

** The 9-B course in algebra at East High School does not parallel the 9-B course in the Rochester junior high schools. East High School is still organized on the 8-4 plan and not on the 6-3-3 plan. In the junior high schools, pupils entering the 9-B grade from the 8-A grade have completed the following work in algebra: the use of letters, the use of the formula, the making of formulas, simple equations and problems, the graph, and signed numbers. At East High School, all this work has to be done in the 9-B semester. This difference in the courses given in the two types of schools is mentioned because the final examination printed here is obviously not similar to the one used in the Rochester 9-B junior high school course.

by the type of question asked on the cumulative departmental tests. Additions were made, of course, to cover the work done at the end of the term for which no departmental tests were given.

Construction of Semester Test—The test was intended to measure comprehensively the central objectives of the 9-B course, and the credits were assigned with due regard for the relative weight of these objectives in the 9-B course.

The central objectives considered, and the weight assigned to each in the examination, may be inferred from the following tabulation:

		Credit
1. The language and the ideas of algebra		4%
2. The formula		
1) The use of formulas	9%	
2) Making formulas	11%	20%
3. Fundamental principles and processes		
1) Addition	2%	
2) Subtraction	2%	
3) Multiplication	7%	
4) Division	7%	
5) Addition and multiplication	2%	
6) Combining terms and removal of parentheses	4%	24%
4. The equation		16%
5. The graph		8%
6. Problem solving		
1) Geometric	7%	
2) Number	7%	
3) Business	7%	
4) Motion	7%	28%
		100%

It will be noticed that the *functional viewpoint is emphasized to the extent that 72% of the credits assigned are given to questions involving functional relationships.*

Method of Tabulating the Results—The plan to make this follow-up study of the semester examination was discussed by the group of teachers concerned, a short time before the end of the term. It was

agreed that such a "follow-up" was desirable, but definite plans were announced only four teaching days before the final examination was actually given. The teachers were provided with an answer sheet for marking the papers, and with a definite instruction sheet for reporting results. Teachers were asked to report a question correct only if the answer was *entirely* correct, except in the case of a question with several parts; here they were requested to report by parts and to report a part correct only if that part was entirely correct. In assigning credits, teachers were allowed to assign partial credit in the case of examples 5, 9, 12, 13, and 14. The question analysis was reported by recording the number of "attempts" followed by the number of "rights" for each question. In computing the per cent correct, however, in the third column of the question analysis, the total number of pupils was used as a divisor rather than the number of attempts.

Analysis of Results—The 274 pupils who took this examination represented the enrollment of *ten* classes. Because of the recent redistricting of East High School pupils, the student body of that school is now largely derived from foreign-born stock. The school history of the pupils was as follows: 173 were new entrants from the 8-A grades of Rochester grammar schools, 15 were from parochial schools, 12 were from out-of-town schools, 14 came from the vocational course in the junior high schools, and 60 were left over from the September 1929-January 1930 entering class of 378 pupils in the academic-technical group and were repeating the subject. One class was composed of "repeaters," while the remainder of the repeaters were distributed in eight different classes. One class of pupils was made up of selected students and included no repeaters.

The medians obtained on the semester test by the ten classes were 83, 83, 82, 78, 73, 73, 71, 70, 68, and 67, respectively. There were no 0% papers and only four 100% papers. The spread in medians was 16% only. The "failure group" obtained a median of 78%. The departmental median was 73%.

The semester test, together with the question analysis, follows:

EAST HIGH SCHOOL

9B ALGEBRA SEMESTER TEST

June, 1930.

Answer all questions. Show all work except that done mentally.
Do not write on this paper.

Part I

	Credits	A	R	% C
1. Copy and complete the following:				
(1) An algebraic expression of three terms is called a	[1]	270	204	74
(2) $7x^2$ and $4x$ are ..... terms.	[1]	273	234	85
(3) In the expression $3x^4$, 3 is called the ....., and 4 is called the	[2]	273	242	88
2. Combine $3a + 7b - a + 4a - 5b$.	[2]	272	253	92
3. Multiply $-3x^2(2x^2 - x - 5)$.	[2]	274	231	84
4. Divide $12n^3 - 3n^2 + 9n$ by $3n$.	[2]	273	230	84
5. Remove parentheses and combine the resulting terms: $7a - 3(2a + 5) + (a - 8)$.	[2]	273	196	72
6. Solve $6n - 2 = 40$.	[2]	272	244	89
7. Solve $\frac{x}{3} + 4 = 11$.	[3]	273	210	77
8. Solve $4x + 7 + 5x - 1 = 6(x - 15)$.	[4]	274	147	54
9. Solve $12a - (9 - 3a) = 4(6a - 9)$.	[4]	258	165	60
10. Multiply $x^2 - 3x + 2$ by $x + 3$.				
Check, making $x = 4$.	[3, 2]			
	M	264	196	72
	C	257	165	60
11. Divide $20a^3 - 22a^2 + 31a - 15$ by $5a - 3$.				
Check, making $a = 2$.	[3, 2]			
	D	263	196	72
	C	247	158	58
12. The dimensions of a rectangle are w and $3w - 6$. A second rectangle has the dimensions $w + 5$ and $3w - 8$.				
(1) Find the sum of the perimeters of these rectangles.	[2]	267	187	68
(2) Find the sum of the areas of these rectangles.	[2]	265	179	65
(3) By how much does the area of the second rectangle exceed that of the first?	[2]	260	150	55
13. Using the formula $P = ahd$, find P if $a = \frac{1}{144}$, $h = 72$, and $d = 62.4$.	[2]	267	195	71
14. Using the formula $F = \frac{Wv^2}{gr}$, find F if $W = 3$, $v = 48$, $g = 32$, and $r = 4$.	[3]	263	159	58
15. Prove by substitution that $b = 4$ is the correct root of the equation $2(b^2 - 3) - b(5 + 2b) + 26 = 0$.	[3]	252	170	62

Part II

	Credits	A	R	% C
16. Write the expression which represents the perimeter of a rectangle whose width is w inches, and which is 6 inches longer than it is wide.	[2]	270	193	70
17. Write the expression which represents the simple interest on \$2000 at 2½ per cent for t years.	[2]	266	155	57

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	Credits	A	R	% C
18. Write the formula for the volume of a rectangular solid.	[2]	267	217	79
19. Using the proper formula, find the volume of a cylindrical gas tank which has a diameter of 50 feet and is 75 feet high.	[2]	267	106	39
20. Complete the following table and write the formula showing the relation between l and w.				
If w = 1 2 3 4 12 ; then l = 2 5 8 11 41	[1, 1, 3]	273 273 273	242 237 235	88 86 86
21. Solve $S = 2\pi rh$ for h.	[2]	264	115	42
22. In the year 1920, a certain stock had an average value of \$100 a share. Since that time, the value of this stock has increased at an average rate of \$10 per share each year.				
(1) Make a table showing what the value of this stock was for each year, starting with the year 1920 and continuing through 1929.	[2]	274	254	93
(2) Draw a line graph, using the table made in part (1).	[4]	274	249	91
(3) Indicate on the graph what the value of the stock was for 1928. Also indicate on the *extension* of the graph what price may be expected in 1933, if the value of the stock continues to increase at the present rate.	[2]	273	240	88
Part III				
23. The second side of a triangular lot is 200 feet longer than the first, and the third side is twice the second side. It requires 1100 feet of fence to enclose the lot. What is the length of each side?	[4, 3]	274	212	77
24. A man bought the same number of 1c, 2c, 5c, and 10c stamps. The amount paid was \$1.80. How many of each did he buy?	[4, 3]	271	183	67
25. Find four consecutive even integers whose sum is 500.	[4, 3]	272	199	73
26. A steamer leaves its dock and travels at the rate of 16 miles per hour. It is followed 30 minutes later by a motor boat travelling at the rate of 20 miles per hour. How far from the dock will the motor boat overtake the steamer? (Solve by algebra or by graph.)	[4, 3]	262	134	49

NOTE A = Total Number Attempting.
R = Total Number Right.

$$\%C = \left(\frac{R}{274} \times 100\right)\%$$

CONCLUSION

The objectives which were included in this test have been emphasized in the 9-B courses for the past three or four years. The results obtained here show some improvement in comparison with those obtained in former semesters. Such an improvement, in this day of the tremendous growth of the high school population and of the consequent lower ability levels, is certainly encouraging. It will be

observed that in these tests an attempt was made to give greater prominence to the *functional* viewpoint, and the continued emphasis on this aspect of the work is beginning to bear fruit. In particular, there is some indication that the average pupil is improving in problem solving, and that the teachers are learning to create a different perspective *while not ignoring the skills of algebra.* There is some evidence that *better results have been obtained both in the skills and also in the things* which mathematicians consider important. Hence the writer of this paper and his colleagues are convinced that the present policy represents a step in the right direction. The most authoritative psychological doctrine maintains that skills cannot be cultivated in a vacuum, but that they derive their significance and educational importance solely from the background that gives rise to them and from the educational pattern into which they are to be built. The time available for the purely mechanical habituation of the skills in algebra is so limited at present that a purely mechanical method of approach seems hopeless.

In conclusion, there appears to be sufficient evidence that *it is both possible and desirable to build skills into a functional situation.* It may be asserted with confidence that teachers who have once given a fair trial to this approach and to this type of departmental testing would not return to the former methods of teaching and testing. This paper is an attempt to show the results obtained from a testing program and a teaching method based on that principle.

Don't Forget!

To mark the ballot on page 63 of the January TEACHER and send to the secretary, Mr. Edwin Schreiber, 434 W. Adams St., Macomb, Illinois, before February 20.

Consumers Uses of Arithmetic Again

By A. O. Bowden

President New Mexico State Teachers College, Silver City, N.M.

The article written by Professor C. N. Shuster and printed in The Mathematics Teacher for March, 1930, which purports to be a "review" of my dissertation, "Consumers Uses of Arithmetic," calls for a reply. The "review" is more of a defense of the vocation of mathematics teachers than a thorough, unbiased review of a piece of educational research.

Professor Reeve, editor of The Mathematics Teacher also commented in an indirect way on the theory involved in the dissertation by quoting and discoursing on the clippings from the New York *World* of May 19, 1929. This was published in the same magazine under date of February, 1930 (p. 71-). Also Dr. Roth of the University of Wisconsin contributed an article in criticising the dissertation and this was published in the December, 1930, number of the same magazine. I wish to comment briefly on each of these articles in this writing.

No book or monograph on any subject ever gets a one hundred percent endorsement from an unprejudiced reviewer and an author is probably wasting time to take notice of all adverse "reviews" of what he writes. But when a reviewer unconsciously or otherwise leaves an impression in his review of an author's book which is misleading and calculated to give a wrong idea of that author's theme, it should not go without challenge.

First, Professor Shuster says, "As a result of this voting, Dr. Bowden attempts to find an objective in arithmetic for the present and coming generation, by discarding all problems that receive less than 25% of the votes". One who will take the pains to read carefully my dissertation will see that no "voting" was undertaken, no opinions or choices were sought. By using the words, "vote" and "jury" he leaves entirely the wrong impression. Voting implies choice between two or more things, persons or situations. People who filled out the blanks in the study were not asked to choose, to vote, nor to give any opinion. They were asked merely to check

type problems which they *had used* since leaving school in life situations aside from their vocational requirements. Second, contrary to his statement, it is not too difficult to find copies of all arithmetics which have been in use within the last twenty five or fifty years.

If one desires to know what kinds and what amounts of arithmetic now in use in adult populations, particularly those who have children in the sixth grade in school he will make inquiry of men and women who are now 43.3 years of age (see p. 36 of dissertation). Men and women of this average age were normally in the elementary schools of the country thirty to thirty five years ago. They studied arithmetic books that were in use thirty five years or more ago. They use *now* some of the arithmetic which was taught them thirty five years ago. In all probability, they do not use *all* the arithmetic which was taught them at that time, but it is safe to say that whatever arithmetic they now use was learned a quarter of a century or more ago. This *does* furnish us a basis for knowing the present value of what they learned of arithmetic three decades ago.

The reason for comparing the content of texts at that time with the widely used texts at the present time is to determine the similarity or difference in the content now offered children and that which was offered these men and women who were children thirty years ago. Professor Shuster seems quite concerned about the number of arithmetics used in this investigation. This collection of text books was intended only to furnish a basis for determining the *type* of arithmetic taught then and now used and the rather close agreement between the kind of arithmetical principles now taught and those taught thirty years ago.

Professor Shuster tries to reduce to absurdity the problems which I used on the blanks on which data were collected for this study. Let us follow his advice and compare the type and variety of arithmetic problems used then and now. The material dealt with in the earlier books *is* different from most modern books but the modern text book writers seem to feel that most of the processes used then should be used now, such processes as extraction of roots, difficult operations in percentage and the like. The problems in modern texts *may* be closer to life than those used thirty five years ago but the variety of processes are painfully similar. Except for space needed to do so, it would be easy to answer each item of his attack (defense), but we take only a few instances from his article for consideration. There

are some thirty million children in the public schools to day. Obviously no single arithmetic text, however popular, is studied by more than a small fraction of this number of pupils. In order, therefore, to secure samples of problems now taught throughout the United States, one must consult all the books in use to get the complete range of problems. Certainly no arithmetic today contains all the absurd problems represented in the dissertation blank list of problems. However, one can truthfully say that children in the American Public Schools are being taught problems of similar nature involving principles basic to the solution of the problems listed in the dissertation.

Below, the problems copied are among those stated in Professor Shuster's article and taken from my dissertation.

246. What must be paid for 6% bonds to realize an income of 8%?

258. The interest for 5 yr. 8 mo. 24 days at 6% is $28.38; what is the principal?

269. What is the difference between simple and annual interest on $1,000 for 5 yr. 6 mo. at 7% payable semi-annually?

For problems involving essentially the same principles as are represented in the above examples see:

Thorndike Arithmetics, Bk. III pages 67-92 inclusive. Revised, 1924.

Standard Service Arithmetics, Grade Eight, pages 82-83, also pages 129 ff., (pub. 1928).

281. 4 is the ratio of what number to 13?

282. 2¾ is the ratio of 23⅜ to what number?

Problems involving essentially the same principles may be found in Hoyt & Peet, Advanced Arithmetic p. 246-47; also The Standard Service Arithmetics, Grade Eight, p. 322-336.

Thorndike, Bk. III; 293-298.

Professor Shuster makes a rather facetious comment on problem 272:

Find the date when due, bank discount, time of discount, and proceeds of a note of $2,400 dated Oct. 19, 1876, due Jan. 1, 1878; with interest at 8% discounted July 26, 1877, at 10%.

He says that "We may be fairly sure that this problem was taken from a book written before 1876 or 54 years ago." Here he is wrong. The book from which the principle involved in such a problem was copyrighted in 1919. It may be said here that many of the problems contained in my dissertation list were copied exactly from no book.

They were made up in such a way that they would include the principles presented to the children in America now and I challenge him to make any successful defense of his facetious remark quoted above. I tried to find out from the companies publishing the books which I used in my study just how wide a sale each had in the respective arithmetics but of course that is a matter on which they would not give out information. If I had had this information I could more nearly tell just what kind of arithmetic was taught to the vast majority of children at the present time, but lacking this important knowledge I had to take examples of the type of arithmetical content from *all* the books used within the last thirty five year period in order to know the kinds offered.

Again, Professor Shuster says: "It seems rather arbitrary to assume that all problems marked by less than 25% of the 682 ordinary parents on the basis of *remembered use* should be discarded. By this same criterion a very large proportion of all the subjects we are now teaching would most certainly be discarded." Quite true, but that is beside the point. Professor Goodwin Watson says in the October 1930 number of *The World Tomorrow* in article entitled "What is a Modern High School?" that one important law of educational science generally accepted shows that most things learned in school are forgotten in a month or so if not used. But what if it is true that most of the material now offered in the public schools should be discarded on the principle of nonuse? That is a rather sad commentary on our ability to select the content of the curriculum, and it is a splendid argument for discarding much of the content now taught in schools. Why not give children something that they *can* use? There is plenty of useful material which the schools are not recognizing. Incidentally he is touching here the main point of my thesis: "Discard the useless and substitute the useful in our schools."

The much quoted statement that I recommended a reduction of the content of arithmetic to 15% of that now taught has been very much misrepresented. On page 27 of my dissertation I make this comment: The four fundamental operations of arithmetic; addition, subtraction, multiplication, and division of simple whole numbers were assumed as universally used in some form, in some degree at least, by every normal literate adult and hence are not included in this study.

In sending out the full list of 345 problems representing all the dif-

ferent types of arithmetic contained in any of the texts which had been taught within the last thirty five years, I felt that a personally supervised checking would be much more dependable than that obtained by broadcasting these among several thousand unknown adults. So the larger list, the original problems were checked very carefully under supervision. The lack of use as represented by the results of this checking was so uniform that I selected only those which had been used. It was from this highly selected list that the shorter list was made up and furnished the basis for my conclusions. I am quite sure that if a larger list had been sent out broadcast I would have obtained the same or similar results. Consequently there was nothing arbitrary about any part of the procedure. The whole point of my recommendation is that we should have very thorough drills in the four fundamental operations with whole numbers to the degree that these processes become as nearly automatic as possible. Beyond these four fundamentals a thorough drill in the use of simple fractions, including decimals, a definite set of techniques in the use of compound denominate numbers should be mastered, an understanding and a developed ability in the use of the simple processes of percentage, interest, discount without time should be the chief emphasis of the efforts of the public elementary schools in arithmetic. All else now taught in the subject of arithmetic is largely a waste of time, money and effort. When this is done much time, interest and effort on the part of the pupils will be saved for other more valuable subject matter.

Quoting further from Professor Shuster's article, "Many of these problems were practically obsolete fifty years ago, others were retained in the text books but seldom or never taught. Practically all the 345 problems on this list can be found in arithmetics published prior to 1850, while on the other hand, there are none of the modern type of problems found in all good recent texts." It would be interesting to know by what method he has been able to find out what is being taught in the schools of the United States. The fact is he does not know. He guesses. None of the books which were actually used and from which I secured my type problems were copyrighted prior to 1919. Other earlier texts were consulted, but it was not necessary to go beyond that date for types represented in my list and those also found in the whole field of elementary arithmetic. It is true that many of these problems were in books published prior to 1850 but they have been continuously retained even in texts now in wide use in the

schools. And this is precisely my contention and criticism. He seems to be unable to distinguish between specific problems and *type* problems. I have meant by "type," and explained my meaning throughout my discussion, not the specific problem but the *kind* of principles necessary to be taught to solve the problems listed. It would not have made any difference whether I selected problems in books printed a thousand years ago so long as I selected problems the solution of which required the principles involved in the books used today. Those who filled out the blanks were not asked to solve the problems nor were they asked to say whether they used the specific problems listed. They understood what I meant by type, too. They *were* asked to check those problems which represented the type of arithmetic now used (See Dissertation, p. 67).

Again quoting from Professor Shuster's article, "It is so palpably unscientific that it will not fool any person who has had any real training in education, but many editors and others without this training have accepted the conclusions of the study and have launched into tirades against arithmetic that have done considerable damage." Yes, these editors have done considerable damage to whom? To those who desire to keep unselected and unreasoned tradition on the throne. But they have not damaged the children of the American Public Schools who are now required to waste much time in learning these problems which are obsolete everywhere except in the text books of arithmeticians and now used in the schools. These wild-minded editors have done damage to some devotees of arithmetic but the schools do not exist for these nor for the subject of arithmetic.

One of the stock ways of attacking research which goes counter to traditional practices or to emotional thinking, or research which shows up inefficiency in vocations is to brand it as unscientific as though there was some mysterious, magical thing about research that is scientific. Many people talk, just as some of these scientists, as though a scientific quality was inherent in certain particular kinds of matter or situations. Science is only a method of finding out the facts. Exact workable knowledge about any situation is scientific. The primitive Indians of America had a lot of scientific knowledge, accurate information, of how to track and kill game.

The dissertation under attack goes counter to the traditional notion of the use of arithmetic. This tradition is a teaching profession tradition which is one of the most difficult types of tradition to oppose.

Tradition is unreasoning and arbitrary and when attacked flounders around for a basis for justification. It opposes changes, first, for one reason then another. Tradition comes into vogue by accident. Some which come into vogue by the laws of chance are good, some are harmless and some are positively injurious. Some that serve a purpose at one time get in the way at other times. Whatever traditions that are serviceable should be kept, but when found to be useless should be discarded.

The body of this paper was written before the appearance of Dr. Roth's criticism which was printed in the December 1930 number of the MATHEMATICS TEACHER. Consequently time and space is not left for a proper reply such as this criticism of his deserves. It so happens that in a general way many of his criticisms have been answered in the above discussion.

There are admittedly some small typographical and mechanical errors in the dissertation such as may be found in most books and writings of a technical nature. No one knows of these any more thoroughly than the writer himself. I make no defense of these and am quite glad to have these pointed out. But when Dr. Roth attempts to condemn the whole theory, method of attack and conclusions and to question the accuracy of the report of data by a subtle type of innuendo he compels one to reply.

On the whole, however, the theme of the dissertation is clear, trends of use and non-use are statistically indicated and minor errors do not completely invalidate the larger conclusions. Statistics is only a short method of description and a way of showing trends. The statistical treatment of any subject never can tell the whole truth. In fact no conclusion based on statistical manipulation, however, mechanically accurate is ever quite true, but the *trend* may be rather accurately indicated and that trend is all that this dissertation is proposing to show.

Some of his criticisms are so palpably unimportant that they need only a passing notice. On page 468 of his article he tries to show that there is probably a great difference in rural and city populations in the use of arithmetic, thus leaving the impression that arithmetical content recommended will not meet the needs of those who live on farms. Some of his criticisms are impossible. For example on page 470 he tries to make up a bit of mysticism in my use of the definitions of kinds of arithmetic. Most of his discussion here ap-

pears to be mere quibble. I tested out the definitions on numbers of people before hand to check up on the point of being understood. Again, on the same page (470) he branches out into a philosophical discussion of appreciation. One may say to this that appreciation of anything or of any subject on the part of any one is a matter of degree. No one can ever completely appreciate the whole of any subject. His suggestion on page 471 that the problems should have been weighted as to their importance would have been arbitrary, indeed, the very thing he criticises when I chose to use 25% non-use as the higher limit in excluding problems from my list.

Professor Reeve quotes at length from the New York *Morning World* of May 19, 1929 as an introduction to his article in the MATHEMATICS TEACHER for February 1930 (p. 71—) on the subject of "The Universality of Mathematics". While he does not offer any direct criticism of the dissertation under fire he does quote at length newspaper comments and opinions of prominent people on the subject of mathematics. I do not know his reaction to the findings of my study. In giving these quotations and his comments he gives me an opportunity to answer certain very wide spread criticisms of the general theory involved.

No one who has had any very great amount of mathematical training or who has studied in the field of philosophy will question for a moment Professor Reeve's contention that mathematics, the science of number, quantity, measurement, form, space has a very important part in the understanding and advancement of almost any of the divisions of science or of business, commerce and industry. Of course number is important for everybody, but an extensive knowledge of and skill in the use of number is not needed for the common school child. Whether he will need much when he becomes an adult depends upon his education after he leaves the elementary schools and the type of vocation he enters. The study represented by this dissertation is in no way touching vocational mathematics whatever. It is not the purpose of elementary education to prepare for any vocation. It has not done so, it does not now do so, and from the nature of our culture cannot do so in spite of the fact that Dr. Roth thinks otherwise. Vocational mathematics should be studied at the time vocational training is undertaken, not before. The discussion in the MATHEMATICS TEACHER for February 1930 also misses the point on page 75-76 when it contends that the problem is not so much one of

content as it is of method. Now, all the methods in the world cannot change the content or amount of arithmetic one needs to use. I quite agree that mathematics teachers as well as those in other subjects from the kindergarten through the university need to know something about methods of instruction—much more than they practice at least, but once the processes are learned by whatever method, they may and will be used when needed. The fact is, not much is needed by the average man. Methods in school are important in the time saved, not in manufacturing content or situations of afterlife where materials may be used.

As to a knowledge of the significance of number, one does not acquire these generalized concepts of number only, and sometimes scarcely, from mathematics books or from classes in arithmetic. He acquires them often quite largely from his general reading of quantity, of space, of time, of height, of depth in literature in other subjects such as history, geography and general science. Also his experience with the world about him and the materials which he handles cannot help forcing these general concepts upon his mind. Now, these technical concepts of number etc., may be acquired only from the study of formal mathematics, but that is beyond the elementary school. Besides, the arithmetic which I have found useful in this study which is attacked, if properly learned will furnish all the feeling, conceiving and appreciating of number which the elementary schools can give.

It is difficult for most people to keep from imputing to many subjects in the elementary school and particularly in the high schools an amount of formal discipline which has never been proved to exist. In this assumption there is a great deal of inexact thinking which the mathematicians so greatly deplore. Many of these inexact thinkers are these very mathematicians themselves. They may be very exact when solving abstract mathematical problems in books, but when they get into other fields such as in history, social science, social values and philosophy they often show many weaknesses in this respect.

We have no quarrel with some of the contentions contained in Professor Reeve's article except only to point out that his contention as expressed there scarcely touches the elementary school child in his relation to elementary arithmetic. It is a waste of his time and that of the teacher to spend much time covering a wide range of mathematical concepts in order to make him philosophical minded

regarding number. All that the child does and can get of this kind of thing may be amply provided for in thoroughly learning the useful part of arithmetic which I have pointed out. Then if the child should be so fortunate as to continue his education through college he may have opportunity to make extended study into the philosophical implications of number, provided his interest has not been killed or his mind surfeited by unholy attempts on the part of teachers to crowd his mind with mathematical concepts beyond his experience to comprehend. The whole point of discussion must not get away from the fact that we are dealing with elementary children and not philosophers and the vocations in planning for the elementary school and concentrate on giving the child as much of the sum total of universal human culture as his mind will digest and leave more complicated materials for later academic years. If the child never goes beyond the elementary school he will have most of the workable concepts in all the fields of human knowledge and can adjust himself in adult life to his social environment.

Finally, the quoting of opinions of mathematics teachers, judges and commencement orators proves nothing regarding the actual value of mathematics in our common life. Opinion is opinion and nothing else and those who indulge in such quotations to prove a point are guilty of inexact thinking the opposite of which mathematics is supposed to give them most excellent training. Neither will being facetious nor making clever remarks nor branding one as unscientific as Dr. Roth and Dr. Shuster have done convince one of the truth or falsity of a proposition. One is led to believe that the dissertation under attack must have value, else it would have been ignored by experts and would not have created all this storm in the press and in the academic world.

Notice to Subscribers

Changes of address for the October number of THE MATHEMATICS TEACHER should reach the office of the Editor not later than September 1, 1931. Subscriptions expiring in May which are not renewed before September 15 will not be included on the October list. Expiration dates of subscriptions are printed on the outside wrapper of each copy of THE TEACHER.

On the Computation of Square Roots*

By L. E. WARD
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THIS paper is intended not only to exhibit a method for computing the square root of a given positive number, but also to give by purely algebraic reasoning a rigorous proof of the correctness of the method, a feature which is sadly lacking in numerous discussions in some of our elementary text books. No originality is claimed for the method.

The various processes for obtaining the square root, or any root, of a given positive number a divide themselves into two classes. In the one class are the methods which make use of a machine, a numerical table, or a tool such as a slide rule; and in the other class are methods which require only knowledge of a process and writing materials. Into the second class fall the method of arithmetic, the method in which the equation $x^2-a=0$ is solved by Horner's or some similar method of successive approximations, and the present method.

Let x_1 be any positive number. In practice it is well to take as x_1 as close a value to $\sqrt{a}$ as we know, but the method is valid whatever positive number x_1 may be. Let $\delta=x_1-\sqrt{a}$. Then $x_1-\delta=\sqrt{a}$ and $x_1^2-2\delta x_1+\delta^2=a$. We shall now discard the term δ^2 in this equation. This leaves a linear equation $x_1^2-2\delta' x_1=a$, whose solution is $\delta'=(x_1^2-a)/(2x_1)$. The number δ' is an approximation to δ. The accuracy of this approximation depends upon the difference between x_1 and $\sqrt{a}$, being better the nearer x_1 is to $\sqrt{a}$. In view of this fact we should expect that the number $x_1-\delta'$ would be a better approximation to $\sqrt{a}$ than x_1. Accordingly we write $x_2=x_1-\delta'=(x_1+a/x_1)/2$, and $x_n=(x_{n-1}+a/x_{n-1})/2$, $n=2, 3, \ldots$. Thus we obtain by the operations of division and addition a sequence of numbers, x_1, x_2, $x_3, \ldots$.

*This paper was presented at the fifth annual conference of teachers of mathematics at Iowa City, Iowa, October 10, 1930, a gathering at the University of Iowa sponsored by the Department of Mathematics and the College of Education cooperating under the Extension Division of the University. The

purpose of the conference is to provide an opportunity for the high school teachers of Iowa to discuss their professional problems with one another and to learn from invited speakers how the latter are solving their teaching problems.

The above discussion is to be regarded as heuristic. We present next several theorems which culminate in the statement that however small a positive fraction be named beforehand, all the numbers of the sequence $x_1, x_2, \ldots$ beyond a certain point differ from $\sqrt{a}$ by less than that positive fraction.

THEOREM 1. *If $x_1>0$, then $x_2>\sqrt{a}$.*

This follows from the equation $x_2-\sqrt{a}=(x_1-\sqrt{a})^2/(2x_1)$, which was obtained by replacing x_2 by its value in terms of x_1, as given above.

We should note that it follows from this theorem that all numbers in the sequence $x_2, x_3, \ldots$ are greater than $\sqrt{a}$.

THEOREM 2. *If $x_1>0$, then $x_{n+1}<x_n$ for $n=2, 3, \ldots$.*

This is proved by noting that it is equivalent to $(x_n+a/x_n)/2<x_n$, which is equivalent to $0<x_n/2-a/(2x_n)$, and this in turn to $0<(x_n^2-a)/(2x_n)$. But this last inequality is true, since by theorem 1, $x_n>\sqrt{a}$ for $n=2, 3, \ldots$.

From these two theorems it is clear that x_3 is a better approximation to $\sqrt{a}$ than x_2 is, x_4 better than x_3, etc. It remains for us to show that $x_n-\sqrt{a}$ approaches zero as a limit when larger and larger values of n are taken. This we do in proving

THEOREM 3. $\lim_{n\to\infty}\left[x_{n+1}-\sqrt{a}\right]=0.$

We have $x_{n+1}-\sqrt{a}=(x_n+a/x_n)/2-\sqrt{a}=(x_n-\sqrt{a})^2/(2x_n)$.
Hence, dividing both sides by $x_n-\sqrt{a}$,

$$\frac{x_{n+1}-\sqrt{a}}{x_n-\sqrt{a}}=\frac{x_n-\sqrt{a}}{2x_n}=\frac{1}{2}-\frac{\sqrt{a}}{2x_n}<\frac{1}{2}.$$

Hence $x_{n+1}-\sqrt{a}<(x_n-\sqrt{a})/2$,

that is, the error in the $(n+1)$st approximation is less than half the error in the nth approximation. From this the statement in the theorem follows immediately.

In practice the method leads with surprising speed to approximate values of $\sqrt{a}$ sufficiently good for all ordinary purposes. It is the experience of the author that if, for example, one approximation is

correct to three decimals, then the next approximation will be correct to six decimals; that is, that each application of the process doubles the number of decimals which are correct. It should be noted also that a mistake made in the arithmetic in applying this method does not vitiate the accuracy of the final result, but merely increases the number of approximations which must be computed in obtaining any desired degree of accuracy.

As an example let us compute $\sqrt{3}$. We choose $x_1 = 1.7$. Then we find that $x_2 = 1.732$, $x_3 = 1.732050808$, which is correct to seven decimals.

In a similar manner we might develop a method for computing cube roots and higher roots, and establish the validity of the method. We will merely give the formula for computing successive approximations to the cube root of a given number, leaving the details of the discussion and the proofs to the ingenuity of our readers.

If a be the given positive number, and x_1 any positive number, then x_n is to be computed from

$$x_n = \left(2x_{n-1} + \frac{a}{x^2_{n-1}}\right)/3, \qquad n = 2, 3, \ldots$$

It will be noted that in the application of this formula the only operations used are multiplication, addition, and division.

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A Mathematics Teacher Seeks Guidance in Guidance

By G. Hapgood Parks
Weaver High School, Hartford, Connecticut

If I had been asked, until very recently, just why mathematics is included in the curriculum of every school, I believe I would have voiced the sentiment of most mathematics teachers by replying "Why, mathematics is so valuable for its own sake! Every pupil needs experience in mathematics in order to fit himself to solve the problems which he will meet in life." But I wonder just how many mathematics teachers have analyzed intelligently the "courses" they are offering to their pupils, to see just how much of the material which is included can be classified truthfully as capable of fitting those pupils to solve life's problems. I wonder, too, just how carefully these same teachers have examined their own methods of instruction, to see just how closely they parallel actual life situations, to see just how capable they are of insuring the pupils' recognition and appreciation of actual practical values and their successful application.

I am afraid that too many of us teachers of mathematics are altogether too prone to be satisfied with the "mathematics for its own sake" ideal. Most of us teach mathematics plausibly, but how many of us actually *know* each one of our pupils anywhere near as well as we know the contents of the text-book we are using? How many of us know the ages of our pupils, their mental equipment, home conditions, personal problems, interests, ambitions? How many of us, knowing these things, make actual use of them to vitalize the subject-matter of our class work and our assignments? How many of us even think of adjusting this subject-matter to the varied interests and the miscellaneous needs of our pupils? How many of us bother to discover any or all of these vitally important facts and—having discovered them—make use of them to turn disrupting failure into the satisfaction of success, thus placing our subject in good repute in the minds of pupils who are interested in consequence of their success? Wherein stands the greater glory, after all, to be a teacher of *mathematics* or a teacher of *pupils*?

There certainly is but one answer to this final question, and that answer voices exactly the spirit of guidance. Here we find that the individual pupil, with his characteristic abilities and interests and needs, is the center of every assignment and of every meeting of every class. How much more worthy is this emphasis than is the more universal one among us which places all attention on the manipulation of complex fractions, or square root, or factoring, or what not!

Weaknesses in my own teaching methods and in those of other mathematics teachers whom I have observed, as well as in most of the mathematics textbooks now in use, were revealed vividly to me at a recent session of the Harvard Summer School. There I was privileged to study with two of our nation's leaders in the guidance movement, Dr. John M. Brewer and Dr. Richard D. Allen. From these men I caught the guidance spirit and was led to wonder just how the field of mathematics teaching could be made to contribute toward guiding the pupil to that situation in which he will be able better to meet and solve the complicated problems of an increasingly complex community life.

To compare mathematics as a whole with, for example, English and the social studies, in a search for natural adaptability to guidance, proved to be very discouraging. As a result, the question came to my mind: "Just what material can a teacher in any branch of mathematics find, and just where can he or she find it, to assist in instilling the spirit of guidance into the job? Just what help can a mathematics teacher get who has heard of guidance and is in sympathy with its spirit, but who knows little or nothing of its fundamental principles?" I decided to find out.

I went at my task fully convinced that there is almost no subject in the school curriculum more immune from guidance tendencies than is mathematics. Anyway, I spent several days browsing among the books on library shelves and among the periodicals available. The results of my quest were so interesting and so instructive that they weakened my original conviction to a very large extent. I should like to share these results with those readers who may find them of interest or of profit.

I received considerable encouragement from the very first volume which I opened. It was Hall-Quest's "Supervised Study," of which Chapter XII is typically guidance in flavor. The main theme is that the pupil must be taught to "learn how to learn." Pupil

activity and initiative are emphasized in such ways as: (1) making of models for class use and to form a permanent exhibit; (2) contribution of original problems; (3) special reports on topics of special individual interest; (4) collection of illustrative clippings from newspapers and magazines indicating value of knowledge and use of mathematics in everyday experiences; (5) recognition of and emphasis on attempts at proofs by methods not given in the textbook; (6) extra-curricula organization in the form of a mathematics club.

All of the above-mentioned activities would serve to motivate the work as well. Other suggested motivation which is applicable from the guidance view-point is: (1) utility of mathematics emphasized by immediate application to actual situations in the individual pupil's experience; (2) advising mathematics as a mark of high achievement for the average as well as the bright individual; (3) posting for exhibition papers of outstanding accomplishment with emphasis on neatness and importance; (4) placing particular emphasis on individual differences in ability and interest; (5) employing periodicals and mathematical puzzles.

I have gone into considerable detail here in order to introduce the spirit of my search for guidance material in the several works which I examined. Such detail will be impossible, as a rule, in this paper.

Some of the material in the 1929 issues of "School Science and Mathematics" added to our guidance fund as credited to mathematics. G. A. Miller in his article, "On Several Points in the History of Algebra," remarks that "the history of mathematics has a peculiar charm for many students of our subject and many teachers of mathematics have found that occasional references to this history are inspiring." He suggested the consideration in clubs of questions relating to the history of mathematics. He brings out an even more valuable thought when he states that a teacher becomes more sympathetic with his students if he acquaints himself with the great difficulty the human race encountered with views which the students are now expected to master easily. Such a study of the history of mathematics would reveal reasons for the difficulties the students face in mastering the subject. To acquaint himself with the history of mathematics, then, is clearly a case of self-guidance on the part of the teacher.

Practically every article which I examined in this periodical is flavored definitely against such instruction as would stress mere

mathematical knowledge and facility in manipulation. There is stressed very strongly, instead, thinking in terms of quantitative relationships. This is brought out with particular clarity by J. S. Georges in his article, "Functional Thinking as an Objective in Mathematical Education."

He says, "the world is infected through and through with such relationships.—It permeates all thinking." He suggests that the teacher of classes in mathematics should provide opportunity for developing a background of these relationships thus causing the pupils to acquire efficient methods of thinking. Georges summarizes the need for this training very vividly: "The great body of scientific knowledge, a great deal of the essential facts of financial and economic sciences, the endless social and political problems are accessible only to those who can think in terms of functional methods and of functional relationships." Two or three illustrations of such functional relationships which he mentioned may not be amiss here: the cost of a suit is a function of the supply of cloth, labor, transportation, etc.; success is a function of preparation and personality; temper is a function of digestion. And there is certainly no field which is better adapted than is mathematics by natural content and method, to develop proper functional thinking if the teacher is aware of its value and attends to its proper emphasis.

It is repeatedly suggested in these articles that the development of facility in the use and interpretation of graphs, particularly when combined with the scientific method of collecting and manipulating data, is the best method of assuring a workable conception of functional relationships on the part of the pupil. This can be readily done by enlisting the pupils to seek out graphs according to the uses made of them in magazines and newspaper articles and advertisements.

An emphasis which is very definitely in accordance with the spirit of guidance is found in G. R. Ray's article, "Types of Errors in Written Work in Plane Geometry." This article describes a method of handling errors which occur in written work in plane geometry. The method which he describes leads the erring pupil to participate in the correction of his own errors in a manner which should be constructive in the development of better understanding and greater accuracy. I feel that the method is worthy of a summary at this point. A list is accumulated of the common types of

errors which have been appearing on the papers, each error is given a code letter, and a mimeographed list of these together with remedial suggestions is supplied to each pupil. On subsequent papers the teacher indicates each error by the code letter only, thus indicating its type. The pupil is led by this suggestion to make his own correction. The pupil keeps a record of his own weaknesses, striving to shorten this list as much as possible as the school year progresses. The teacher checks the efficiency of the method by keeping a frequency record of the recurrence of the same type of error on the papers of each individual pupil. This is a very excellent application of educational guidance.

M. A. Potter's article entitled "The Drift of Junior High School Mathematics" smacks throughout of pupil-mindedness. It reads, in part: "Make the classroom and the school yard a laboratory, give the pupils tools and lead them to make their own discoveries. They need much direction, but they should be urged to think rather than imitate, to reason rather than memorize." Various cases of pupil activity which illustrate education of a high order are cited. One is particularly interesting. The judge of the court in a certain town was accustomed to send to a distant university for charts to be used in his court cases. A mathematics class in the junior high school of the town supplied him with a bar graph representation of the distances required for stopping autos traveling at various speeds. He made use of it in judging cases of auto speeding, much to the glorification of the class and their teacher. Among other means of provoking interest, the author suggests scrap-books showing cartoons, clippings, original drawings, advertisements and quotations relative to thrift and investments, mathematical phrases discovered in literature, history of mathematics, mathematical jokes, etc. Particular emphasis is placed on interesting the pupil in his own progress, making him aware of his own weaknesses and anxious to correct them.

"Mathematics in Industry" by C. J. Leonard, makes note of a guidance spirit in an evening school where men were studying in an attempt to advance themselves in their positions of employment. He explains that the main cog in the wheel of advancement for these particular men was mathematics; practically every other necessity could be learned on the job. Each man was urged, no matter what classes he was enrolled in, to bring in any mathematics problem which might be interesting him, which, at the same

time, was giving him difficulty in its solution. By organizing the cooperation of all of the instructors at the school, solutions and explanations were developed and much of benefit for the men resulted.

The February, 1929, issue of the same magazine supplied me with my first discovery of anything which was definitely and intentionally guidance. It was G. Forster's detailed description of "A Three-Track Plan for Physical Science and Allied Courses" at Pasadena (Cal.) high school and junior college. Here I found guidance from the administrative point of view. Consideration is given to individual pupil differences in ability and accomplishment by a three-track plan in which the pupils of each grade are subdivided within each of the "tracks," provision being made for ability and accomplishment grouping to a very great extent. Another important feature of the arrangement is the paralleling of classes in mathematics with the sciences in which they can and will be applied. The third track takes care that slow pupils have mathematics classes adjusted as nearly as possible to their abilities, and for those who are unable even to complete this degree of mathematics, appropriate classes in science are still available. The program is so arranged that advancement may be either by group or individual promotion. Pupil activity is assured through laboratory work, pupil demonstration, and pupil assistanceships in which capacity the more able pupils help their slower comrades in other groups.

Having completed the 1929 issues of "School Science and Mathematics" I turned with high hopes to the "Vocational Guidance Magazine" files. This field was very disappointing, however, until I found in the November, 1928, number, A. D. Seaver's "A Brief Annotated Bibliography of Guidance Through Mathematics." This article supplied me with nearly a score of references which served to guide the next several hours of my quest. They led me to some very interesting material.

In S. C. Parker's "Methods of Teaching in High School," pg. 381, there is described a monitorial system of teaching geometry which is very much in accord with the spirit of guidance. The class starts the year as a single unit with the teacher in charge, but soon it subdivides into three groups according to ability manifested by its members. The rapid and medium groups are each led by one of its own members in turn. The slowest group is led by one and then

another member of one of the two more rapid groups. Each group progresses at its own speed, working independently in a corner of the room with the teacher sharing time with each as needed. As evidence of the efficiency o' this method, let it be stated that the most rapid group covered plane and solid geometry in one year, the medium group covered five books of plane geometry and the slow group completed four books satisfactorily. It was particularly emphasized that this method of grouping gave every pupil an opportunity to participate regularly.

In the Third Yearbook of the National Council of Teachers of Mathematics, two titles caught my eye as particularly significant. They were "The Fallacy of Treating School Subjects as 'Tool Subjects'," by C. H. Judd, and "Mathematics in the Training for Citizenship," by D. E. Smith. Both of these articles might prove to be inspirational to a teacher, but they were disappointing failures from the point of view of guidance.

Chapter IX of J. W. A. Young's "The Teaching of Mathematics" supplied some of the very best material to come to my attention. His central idea is that the child's main desire is *to do* and the teacher's job is to supply the work, planned in such a way that the pupil succeeds, but still has worked hard enough to feel elated by his success. Emphasis on subject matter is conspicuously absent. Instead, we read of experimentation, estimation, approximation, observation, application, induction, intuition, common sense. Graphs are here emphasized again, but "to illuminate work not to 'teach graphs'." The bulk of the work is done in the class-room where an informal relation exists between teacher and pupil. The teacher is a leader and a friend to the pupils, who are working singly or in groups as conditions demand.

Diemer and Mullen's "Pupil Citizenship" supplied some material which could be applied very helpfully in arithmetic teaching. Examples of citizenship and home membership guidance in the form of problems involving safety statistics, thrift and savings, real situations in road construction, projects involving the family budget, and cooperation with the production of the school paper, the year-book and the management of the school garden were cited. Much of value should result from the use of such material in junior high school classes in mathematics.

Gowin, Wheatley, and Brewer's "Occupations" summarizes the relations between and the uses of the several branches of mathe-

matics, but supplies us with very little of guidance value beyond this information.

E. C. Moore, in "What is Education?" places a great deal of emphasis on pupil activity, reminding the reader that to know how to use a process means *use it*. A worthwhile emphasis which was noted in no other place was brought out by this author. "We must remember," he says, "that doing is mental as well as manual." He describes a good school as a work-shop where, by their own efforts, the pupils learn to use tools under the master's eye.

The most consistently pupil-minded of all of the books which I read was E. L. Thorndike's "The New Methods in Arithmetic." His whole thought is wrapped around this statement: "Every task assigned should fit the child for life and real affairs, to which it applies, should be connected with it in the pupil's mind." The teacher is advised to "study the learner as well as the lesson. Consider the situation as his mind meets it and the tendencies he has toward it as well as the responses you want him to make." This author suggests the use of games, familiar objects at home, and other studies in school to provide reality, to increase interest, to illustrate a meaning, or to apply a process. Several valuable illustrations are analyzed. The spirit of the book is further brought out by his criticism of common methods of teaching. "Teaching tends to prepare for testing," he says, ". . . . The examination that really counts is fifty years long; its situations are real things and events; it means mastery of a few things rather than sixty percent efficiency in many . . . . . Examinations, like explanations, drills, definitions, and rules should be for the learner and for life." This book is particularly rich in materials for the first six grades, but *any* teacher of mathematics would profit by reading it—particularly Chapter VIII.

The rest of my material I am going to classify in two groups. My first group consists of several articles published between 1894 and 1912. Although I found only two among these which were at all helpful I was very much interested in comparing the flavor of these articles, as a whole, with those more recently published. This comparison revealed a very striking tendency toward the principles which leaders in the guidance movement are writing on their banners—a very encouraging situation, I think. But if I may digress from my subject momentarily to mention an observation which has been brought forcefully to my attention I would em-

phasize strongly the fact that, whereas the trend of mathematics teaching as a whole during the past several years has been definitely toward a more wholesome treatment of the subject, no such trend is to be found when we study the examinations to which our college aspirants are exposed by our College Entrance Examination Board. The result of such conservatism on the part of this Board is self-evident: those schools which are supplying their pupils with that brand of mathematics which will be of most value to them as citizens of the world are developing an immense percentage of failures amongst those who face the examinations of this Board. In the largest schools, perhaps, pupil classification will allow of reasonable success in overcoming this obviously unnecessary obstacle, but just think of what it means to the vast number of pupils in the smaller schools! There, all must suffer the rigors of "college entrance" mathematics—where teachers capable of conducting it can be supplied—even at the complete loss of experience in the more practical parts of the field, or else college hopefuls must get their preparation through tutorial channels. Certification is only a very inadequate partial solution of this situation.

My last group is the more interesting one, and, perhaps, the most valuable of any from the point of view of pupil guidance. It includes several volumes and pamphlets which are strictly recreational in nature, material which, I fear, is foreign to the experience of altogether too many of us teachers of mathematics. As of particular interest and value I would list: D. E. Smith's "Number Stories of Long Ago"; G. M. Wilson's "Motivation in Arithmetic"; W. E. Stark's "Measuring Instruments of Long Ago"; W. S. Guiler's "Teaching Arithmetic Thru Games and Other Pupil Activities"; R. Schorling's "Illustrated Mathematics Talks by Pupils of the Lincoln School"; S. I. Jones' "Mathematical Wrinkles"; and H. E. Dundenny's "Amusements in Mathematics."

In summary, let me say that this study has shown me that although there has been a very definite trend during the last several years toward a more satisfactory emphasis in the teaching of mathematics, there is still a great deal to be hoped for. Much more can be done than is being done, and my search for material has shown me that there is plenty of it available for the teacher who is willing to seek it out, cull the best of what is accumulated, and *put it into practice.*

It would require a volume to analyze with any degree of complete-

ness even what material I examined during those few days. The most I can do in this paper is to offer the material generally and let the teacher select what can be applied to the work in which he or she is particularly interested.

Finally, let me say that this study has revealed very emphatically to me that in order to be able best to select that material which is of greatest worth from the enormous amount that has been written; in order to be ready to guard against the misguided influence of over-enthusiastic, mathematics-minded writers; and, what is even more important, in order to be ready to apply with the best results the material which is selected, *the teacher of secondary or grammar school mathematics should certainly make himself or herself well acquainted with the principles of guidance itself, first of all.*

National Council Yearbooks

First Yearbook (1926)—A General Survey of the Progress in Elementary and Secondary Mathematics of the Last Twenty-Five Years. Price $1.00, 210 pages, paper cover. (Out of print.)

Second Yearbook (1927)—Curriculum Problems in Teaching Mathematics. Price $1.50, 297 pages, in cloth, or $1.25 unbound.

Third Yearbook (1928)—The Teaching of Significant Topics in Mathematics. Price $1.75, 276 pages, in cloth.

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Number Contents of Front Page of Newspapers

By Ben Sueltz
State Normal and Training School, Cortland, N.Y.

This report is based upon an analysis of the number content of newspapers published in central and southern New York state. A total of thirty-five copies of fourteen different publications were examined for the numbers that appeared in figures. These thirty-five copies were distributed among the publications so that two or three different copies of each publication were used. In no case were more than five copies of a single publication used. In general there was no forethought of the type of news that would be found in the papers and analysis showed that no event of an extraordinary nature which would be full of figures was chronicled in these papers. The papers were selected by name and date before they were published so that it is very evident that it was impossible to select them because of their contents. These newspapers were published in cities ranging in size from 15,000 to 300,000 population. Chart B shows the list of newspapers used and their frequency of use as well as the approximate population of the city represented.

Inasmuch as this analysis was confined to the front page of the newspapers it covers what is often called "world news". Various papers published on the same date had similar news content and for this reason the numbers appearing in the body of the news were the same. This was especially true of press service dispatches such as the Associated Press dispatches. The numbers occurring most frequently were those used in giving the date of the dispatch, such as Feb. 2, and Feb. 6. The size of the city had no marked effect upon the number content of the newspaper published there. This was true in respect to both the size of numbers and the actual number count of numbers. The range of numbers is especially interesting because its extent and also because there seems to be no basis for predicting what numbers may be found in any newspaper except that they are apt to be most any number between rather small decimals and large numbers in the billions. Therefore it naturally follows that anyone

who wishes to read a front page of a newspaper and do it with any degree of understanding of the number content must be able to comprehend numbers within the limits named. Detailed information concerning the number content and the classification of these numbers appears in an accompanying chart. (Chart A)

CHART A

This chart shows the frequency of appearance of numbers that occurred ten times and more. It lists the numbers in the order beginning with that of greatest frequency.

Number	*Frequency*	*Number*	*Frequency*	*Number*	*Frequency*	*Number*	*Frequency*
2	250	15	51	32	21	27	13
8	189	31	45	917	21	22	12
6	181	24	44	40	21	60	12
7	179	30	40	30,000	18	80	12
1	157	16	37	26	17	500	12
11	157	28	36	1,922	16	10,000	11
3	152	14	36	45	16	33	11
10	121	1,928	30	35	16	34	11
5	120	50	27	25,000	16	38	11
4	104	13	26	5,000	15	1,921	11
12	74	1,929	26	6,000,000	14	1,920	11
1,930	71	17	26	112	14	56,000	11
20	59	100	25	3,000	13	1,000,000	10
9	55	25	24	21	13	1,927	10
18	54	100,000	23	23	13	200	10

Some of the significant and interesting conclusions from this study may be stated as:

1. The actual range of numbers was from .01 to 10,000,000,000.

2. In general, the numbers occurring with greatest frequency were relatively small in size and those of least frequency were relatively large. However, the frequency chart of all the numbers doesn't show that large numbers do not appear rather often.

3. Numbers of four and more places are usually rounded. An outstanding exception to this is the case of four place dates such as 1928 and 1930 which of course would be recorded exactly.

4. The greatest single use of numbers was to show the date of the month. This accounts for the great frequency of numbers below 31.

5. The total number of numbers in the thirty-five papers was 3,917. It is possible that more numbers actually did occur and their presence was not detected by the reader. This shows more than one hundred numbers per front page.

6. The total number of different numbers was 497.

7. A total of 118 decimals appeared and 81 of these were different. The decimal content was about 3% of the whole.

8. Most of the decimals were of one and two places and were of the kind usually referred to as "mixed decimals". However, many "pure decimals" appeared and several four place decimals were found.

9. A total of 64 fractions and mixed numbers of which 26 were different appeared. The fractions constituted about 1.6% of the whole.

10. Thirty-four or a little over half of the fractions had denominators of two. Those using four as a denominator numbered thirteen

CHART B

This chart shows what newspapers were examined and tells how many issues of each paper were used.

Name of Newspaper	*City*	*Approx. Population*	*No. of issues.*
Auburn Citizen	Auburn	35,000	3
Binghamton Press	Binghamton	75,000	5
Binghamton Sun	Binghamton	75,000	5
Cortland Standard	Cortland	15,000	5
Elmira Star Gazette	Elmira	50,000	1
Ithaca Journal	Ithaca	20,000	2
Middletown Times Herald	Middletown	20,000	2
Rochester Democrat and Chronicle	Rochester	350,000	1
Rome Daily Sentinel	Rome	30,000	1
Syracuse American	Syracuse	200,000	1
Syracuse Post Standard	Syracuse	200,000	5
Syracuse Herald	Syracuse	200,000	3
Utica Daily Press	Utica	100,000	2
Utica Observer and Dispatch	Utica	100,000	2

and amounted to about one-third of all the fractions. The figure three occurred six times as a denominator and eight appeared four times.

11. Decimals were almost twice as prevalent as common fractions and appeared in more complex situations and forms than did fractions.

12. One hundred fifty-two numbers were above one million. This group makes up about 4% of the total.

13. No evident regularity of size of numbers was found. Any number up to hundred billion is apt to occur but if a large number does occur it is apt to be rounded.

In summary it is interesting to note that number content is rather prevalent on the front page of newspapers and that these numbers cover a wide range. It is interesting also to speculate about the number content that one would find in a study of newspapers which come from all sections of the country. From the similarity of papers it would seem that perhaps such a study would not show very different results from those found in the present study. Inasmuch as the study made did not include papers from very large cities or metropolitan papers one may wonder what effect their inclusion would have had in the results.

It is not always easy to classify writers into two groups, one strongly favoring mathematics for the training of the mind and the other opposing it. Some authors express only a qualified assent or dissent. Hardly any one ever seriously advocated mathematics or any other single subject of study, as affording by itself alone a satisfactory training of the mind. Hence an author may argue against the training of mere mathematicians, or against the study of mathematics to the exclusion of other studies, and yet assign to mathematics a conspicuous and generous place among other studies in a course of liberal education. We place such authors among those favorable to mathematics.

During these two centuries of great intellectual achievement, questions of method or research in science and philosophy, of mental preparation for conducting investigations and of augmenting intellectual accomplishment received renewed attention.

Philosophers and divines took a leading part in such discussion. Those that opposed mathematics as a leading topic of study in education were Alsted, Bossuet, Buddeus, Gundling, Condillac, Fénelon, Du Hamel, Hobbes, John Newton, Huet, Hume, Poiret, Le Clerk, Madame de Staël, Barbeyrac, Vico,—16 in all. Among philosophers strongly favoring mathematics in liberal education were Descartes, Spinoza, Locke, Barrow, Berkeley, Kant, and Voltaire,—23 in all.

Educators favored mathematics 6 to 1, Basedow being the one opposed. Mathematicians (including Leibniz, Wallis, D'Alembert, Pascal) stood 59 to 1, the one against mathematics being Chevalier de Méré. Scientists who were not mathematicians voted 4 to 1,—John Gregory being against mathematics.

Of literary men and statesmen, 11 (including Cromwell, Franklin, William Pitt, Francis Bacon, Addison, Haller) favored mathematics; while 11 men (Sorbiere, Bayle, Dunton, Warburton, Swift, Desfontaines, Burnett, Gibbon, Walpole, Hyde, Frederick the Great) were opposed to it.

Of the total number of witnesses of the seventeenth and eighteenth centuries that we were able to summon, 103 recognize high value in mathematics for liberal education, while 30 deny such value.

—Cajori, Florian, *Mathematics in Liberal Education*, Open Court Publishing Company, pp. 73-74.

Human Elements in Mathematics

By E. T. Browne,
Associate Professor of Mathematics, The University of North Carolina

Some years ago when the proposal was made in a certain liberal arts college that mathematics be no longer a required subject, the professor of philosophy said to me:

"I have been asked if mathematics should be listed as a prerequisite to my courses in philosophy. To this I can only answer that there is no subject which gives a more satisfactory background for a study of philosophy than a good course in mathematics, provided that course be properly taught. The only trouble is that so many students come out of a course in mathematics with only one recollection of it, and that is 'For Tuesday we will work so many problems on page 53'." This professor of philosophy was profoundly interested in mathematics and was, in fact, a mathematician of no mean ability. My fears are that his criticism was only too just. How can we so teach our courses in mathematics that our students may carry away with them something else than the idea that their course consisted only in "Work so many problems on page so and so"? That will be the subject of my discourse.

The study of mathematics is by its very nature different from many other studies. Not because of the symbolism employed, for this is a superficial aspect of the subject. In fact, for many centuries mathematics was not symbolic, but rhetorical, i.e., all statements were written out in full—not by means of a compact symbolism. In fact, our symbolic mathematics of today is largely the result of the last four or five centuries, and it was this introduction of symbols that freed mathematics from the fetters that enchained it. But the difficulty is more fundamental than that. Mathematics is a *structure*. Each proposition or theorem is built up on propositions and theorems that come before, just as the first floor of a house is built up on

(An address delivered before the Mathematics Section at the Eighth Annual Convention of THE NORTHWESTERN DISTRICT TEACHERS, High Point, North Carolina, Oct. 24, 1930).

the foundations, and the second story is built up on the first, and so on. In most cases it is impossible for one to pick up a book on mathematics and read understandingly page 101 without having previously read the first hundred pages. It is on account of this fact that in my first lecture to my students at the beginning of a new quarter, I urge them to study the assignment carefully every day thus building up a strong foundation. Moreover, just as the mortar and cement between the bricks in the foundation should be allowed to harden before the first floor is added on, so should the students learn a little mathematics every day and assimilate that, or let it *set* as concrete does, before adding more mathematics to that. Hence, by its very nature, mathematics is a discipline that requires unflagging daily attention. This necessitates a certain amount of drill, and it is from this that the student gets his idea of mathematics as "Work so many problems on page so and so".

Yes, the successful study of mathematics does require a certain amount of drill. One cannot read a mathematics book as he would a novel. He should sit down at a table with a pencil and plenty of paper and when a statement is made, the truth of which is not at once evident, he should challenge the statement and by actually working it out, should convince himself of its truth or falsity. In the words of B. F. Finkel "As the drill will not penetrate the granite unless kept to work hour after hour, so the mind will not penetrate the secrets of mathematics unless held long and vigorously to the work. As the sun's rays burn only when concentrated, so the mind achieves mastery in mathematics, and indeed in every branch of knowledge, only when its possessor hurls all his forces upon it. Mathematics like all other sciences opens its door to those only who knock long and hard."

Thus, those of us who are teachers realize that a certain amount of drill is necessary if the student is to master mathematics. But we should not lose sight of the fact that, while drill is necessary, it is by no means all. Failure to realize this may mean that our students will go out from our classes with the remembrance of mathematics as "Work so many problems on page so and so". How then can this state of affairs be avoided? My answer is that the responsibility rests primarily on the teacher. He should have at hand at all times illustrations showing the bearing of mathematics on daily life, or anecdotes from the history of mathematics, or some interesting fallacy

to exhibit. The history of mathematics alone would be sufficient to furnish most teachers with all such anecdotes and incidents necessary. Indeed, every teacher of mathematics should have on his shelves at least one good book on the history of mathematics.* Or, such a book should be on the shelves of every high school library.

In the sciences, one generation tears down what the preceding generation has built up; but in mathematics each generation adds another story to the lofty structure that the previous generations have raised. Nor must we think of mathematics as a static subject. It is constantly growing, as is evidenced by the great theory of relativity as propounded by Einstein during the past generation.

Many students grumble at having to do sums, multiplications and divisions. How many of them realize how much easier it is for them than it was for the Roman youth of two thousand years ago? The introduction of the Hindu-Arabic numerals, 1, 2, 3, etc., together with the concept of local value, that grandest achievement of the Hindus, and the one which, in the opinion of Cajori, of all mathematical inventions, has contributed most to the progress of human intelligence, renders almost childishly simple the problem of multiplying 27 by 48. Imagine the difficulty which a Roman must have experienced in multiplying XXVII by XLVIII. In fact, the only methods that he had for doing this was by tables or by use of the abacus. On account of the difficulty of the process various schemes were devised for rendering the process easier, for example, that of repeated doubling, which is said to have been used by the Egyptians, and until recently by certain peasant tribes in Russia.

To illustrate the Russian method of repeated doubling we place the 27 and the 48 in parallel columns. Under the 27 we place 13, the quotient by 2, the remainder being disregarded. Under the 48 we place its double 96. Similarly, under the 13, we place 6, its quotient by 2, while under the 96 we place its double. We continue in this manner until we reach a quotient of 1 in the 27 column. We then delete from the 48 column those numbers such as 192 which correspond to even numbers such as 6, in the 27 column. The sum

* One of the best and most readable books on the history of mathematics is Ball, *A Short Account of the History of Mathematics*, Macmillan. Another excellent history is that of Miss Sanford, *A Short History of Mathematics*, Houghton Mifflin. Other good histories are those of Cajori, published by Macmillan, and Smith, published by Ginn.

of the remaining numbers in the 48 column will then give the product of 27 by 48.

27	48
13	96
~~6~~	~~192~~
3	384
1	768
	1296 = 27×48.

Thus the process of multiplication is replaced by the process of repeated doubling and halving, a process which could be performed on his fingers, if necessary. The student who sees this cannot fail to realize how much easier these operations are for him than they were for people working by this method.

There is scarcely a teacher who has not at some time or other had some student remark to him "What do I gain by learning this mathematics?" We must not deceive ourselves. There is nothing new in this attitude. It is said that Euclid was teaching a youth geometry and after they had finished the first proposition, the youth inquired, "What do I gain by learning these things?" Whereupon, Euclid called his slave and said "Give the youth three pence, since he must make gain out of what he learns".

Illustrations and incidents such as this can serve in no small way to relieve the monotony of the routine of the class room. Many are found in the history of mathematics. Among others I may mention Euler's classical proof by algebra of the existence of God.

Leonhard Euler of Switzerland was one of the greatest mathematicians of the 18th century. His latter years, during which time he was afflicted with blindness, were spent at the court of Catharine II in St. Petersburg. One winter the French philosopher Diderot paid a visit to the Russian court, and he conversed very freely and gave the younger members of the court a great deal of lively atheism. Thereupon, Diderot was informed that a certain learned mathematician of the court was in possession of an algebraic demonstration of the existence of God, and would give it to him before all the court if he so desired. Diderot consented. At the appointed time before all the court assembled, the blind Euler entered led by a small boy. Euler walked straight up to Diderot and in a tone of perfect conviction

said "Sir, $(a+b^n)/n = x$, is it not?" Diderot, to whom algebra was Hebrew, replied "Yes", whereupon Euler answered, "Therefore God exists! Answer that". Loud peals of laughter broke out on all sides and Diderot, embarrassed, asked permission to return to France which was granted.

Such instances from the history of mathematics may be multiplied indefinitely. Examples may also be cited from other fields. All students are interested in the notion of the infinite. The question of division by zero is also a favorite one. Mathematicians are agreed that the quotient 1/0 does not exist. There is no number to represent this quotient, since there is no number whose product by zero is equal to 1. Indeed, division by zero is excluded from the operations of algebra; or, as Wilczynski has put it, "The thirteenth commandment should be *Thou shalt not divide by zero.*" However, the question of the value of 1/0 never fails to arouse interest. Some students will declare that its value is *infinity*, others say that its value is zero while still others say that its value is 1. Upon questioning the last named group as to why they say the quotient is 1, I have already had the reply: "If you divide 1 by zero you are not dividing 1 by anything, and if you don't divide 1 by anything you have your 1 just as you did when you started. Hence the quotient is 1." Of course, this is pure sophistry, but it is hard to convince the student of that.

No discussion of this sort would be complete without the mention of the three famous geometrical problems of antiquity—problems that for more than 2000 years defied solution, although they were attempted by the greatest minds of the ages, and which only within the past century have been proved impossible. These are (1) the trisection of an angle (2) the duplication of the cube and (3) the squaring of the circle. The origin of these problems is shrouded in mystery. Although there are certain legends associated with them, still it is generally believed that the problems arose naturally when the Greeks were attempting to build up the structure of plane geometry. Certain it is that the problems were considered by the Pythagoreans (500 B.C.). According to Ball, the philosopher Anaxagoras rashly asserted that the sun was a hot rock larger than the Peloponnesus; whereupon he was adjudged guilty of impiety and cast into jail. While there he wrote a treatise on the squaring of the circle.

By the duplication of the cube is meant the construction, by means

of ruler and compasses alone, a cube whose volume shall be twice that of a given cube. It is said that the Athenians in 430 B.C. when suffering from a plague consulted the oracle at Delos as to how they could stop it. Apollo replied that they must double the size of his altar, which was in the form of a cube. The suppliants hastened home and constructed a new altar having its edge double that of the old one, so that the volume was increased eight-fold. Whereupon, the indignant god made the pestilence worse. This story is of course purely legendary. Doubtless the problem arose naturally in the Greek investigations in geometry. It is extremely easy to duplicate the square, that is to construct a square whose area is double that of a given square. In fact, all that is necessary is to take the diagonal of the given square as a side of the desired square. But the duplication of the cube is an entirely different matter. In fact, it was proved conclusively by P. Wantzell in 1837 that the latter problem is incapable of solution by ruler and compasses alone.

By the trisection of an angle we mean the division of the angle into three equal parts by use of straight edge and compasses alone. The student of elementary geometry knows that it is quite easy to bisect an angle. Moreover, he knows that it is a simple matter to trisect a right angle. Why, therefore, should he not be able to trisect any angle? This problem too is impossible, as was proved only within the last century, if we confine ourselves to the use of compasses and straight-edge only. However, it can be accomplished by other methods. In fact, Archimedes about 200 B.C. gave an elegant solution by use of a graduated ruler.

The last, and probably the most famous, of these three problems is the squaring of the circle. That means to construct by compasses and straight edge alone, the side of a square whose area shall be equal to that of a given circle. Every student of elementary geometry knows that it is easily possible to construct a square whose area is equal to that of any given polygon. Is not a circle the limit of a regular inscribed polygon as the number of sides increases without limit? If then it is possible to construct a square equal in area to any polygon, should it not be possible to construct a square equal in area to a circle? The ancient Greeks argued in exactly that manner. But the two problems are essentially different. The one is possible while the other is not. This fact was proved conclusively by Lindemann in 1882 when he showed that π is transcendental, i.e.,

that it is not the root of an algebraic equation with integral coefficients.

It should be of interest to the student to know that there are some insolvable problems in mathematics. After reading some of the intricate proofs and constructions in geometry, he may be led to believe that there is no mathematical problem impossible to the mathematician. Nothing could be farther from the truth. The whole field of mathematics bristles with problems which have not only never been solved, but have never been shown to be incapable of solution.

Although these three famous problems have been proved incapable of solution, attempts are still being made to solve them. These vain efforts are, however, being put forth by men who are really in ignorance as to just what the problem is. Only a few days ago I noticed in the daily papers a news item to the effect that an Italian mathematician had succeeded in squaring the circle. And only a year or so ago I noticed that an army officer in California had succeeded in trisecting the angle.

Other items which would elicit the interest of students are fallacies of all kinds, discussions of the infinite, the account of the discovery of the planet Neptune, etc. In fact, some books on plane geometry include a number of interesting fallacies. Of course, in order to discuss these topics, the teacher must be well read, and on the *qui vive* himself. He should read mathematics books other than the text which he is using, such as histories of mathematics and books on mathematical recreations, of which Ball's classical work* is an excellent example. This gives the teacher more work, but that is the price we must pay. One who is making the teaching of mathematics his life work should not begrudge that price.

* *Mathematical Recreations and Essays,* (Macmillan). Other excellent books are Schubert's *Mathematical Essays and Recreations* (Open Court), White's *Scrap Book of Elementary Mathematics* (Open Court), etc.

On To Detroit!

Values of Primal Mathematical Concepts: a Picture

By FLETCHER DURELL

Belleplain, New Jersey

THE PROGRESS OF mathematics during the past one hundred years or so has been characterized by two principal movements.

The first of these tendencies has been the enormous development of pure or abstract mathematics in various directions. Some of these extensions of the pure technique of the subject have already found special fields of application as in the subjects of relativity, wave mechanics, the quantum and electron theories; or in general in what are termed the New Physics, and the New Astronomy.

The second of the two movements under consideration is what has sometimes been termed informational mathematics, but which may perhaps be more adequately described as primal, cultural mathematics. One item of this has been the development and wide use of the statistical graph. It is also illustrated by the Dewey method of catologuing books, and, in like manner, ideas and objects in general, by the use of the Arabic number system.

The second of these movements when fully developed should prove to be of equal if not greater value than the first. The results of the first are directly usable only by a small number of special workers in a few departments, while both the spirit and details of the second movement can be utilized by everyone, every day, and many times every day. Fully developed and employed the latter of the two movements means a grasp of the spirit of mathematics in such a way as to give a perspective and new meaning to all of life.

It is the object of the present paper to give in a somewhat pictorial or narrative form, some illustrations of the general cultural values of the elementary mathematical concepts. With this end in view we will suppose that a traveler, equipped with a good grasp of these primal mathematical ideas, goes to Egypt, the fountain-source of our civilization and we will then note the possible uses which he might make of these concepts in getting the utmost value and meaning out of his observations and experiences.

Hence in what follows it is important that the reader should carefully note each use of the categories of number, size, position, symbolism, dimensions, ratio, and so on, and also the various ways in which these are utilized as aids in forming a definite, organized picture and in extracting full meaning and value from the same. Especially would we call attention to what Emerson terms "the inexhaustible expressiveness of form."

Our traveler observes that the land of Egypt taken as a whole may be regarded approximately as a narrow rectangle with a length of about 700 miles or three-fourths of the distance from New York to Chicago; and an average width of about 10 miles. Its soil is from 40 to 50 feet deep. Hence so far as its fertility is concerned it may be said to be a three dimensional country rather than an area.

The flood of the Nile begins late in June and rises to a height of from 20 to 28 feet. It affords the most reliable and best controlled water supply of any large agricultural region in the world.

The land produces from 3 to 4 crops per year and is now selling for $2000 per acre.

Besides these unequalled agricultural resources, Egypt is also the focus of certain other advantages, some of them geographical in nature, which might be pointed out. Hence it is not surprising that this country was the starting point of civilization as we know it.

The Pyramids and Sphinx

As the traveler proceeds from Cairo out towards the great pyramids at Ghizeh he notes that the flat and narrow surface through which the river Nile flows to the North is bounded both on the East and West by bluffs about 60 feet high, formed by rocks partly covered by sand which has encroached from the desert that stretches away to the horizon.

On the western bluff, two or three hundred feet back from its top, is situated the Great Pyramid of Cheops. The base of this is a square whose area is 13 acres, and the height of the pyramid is approximately 20 per cent less than that of the Washington Monument. The erection of this pyramid required the labor of 100,000 men for 20 years. This huge structure was built to serve as the tomb and for the glorification of one man, a king and a despot. It is a symbol of human selfishness and tyranny.

In front of the great pyramid, the Sphinx has been carved out of the very rock which constitutes the bluff. The sphinx consists of

the body of a crouching lion over 180 feet long, and a human head rising to a height of 60 feet. It has its back turned to the great pyramid and its face looks toward the East; that is, out over the fertile plain of Egypt toward the rising sun.

On the pyramid every line is straight and every surface flat. As a result the huge object expresses static strength, resistless immobility. If it also expresses aspiration, it does this in a local and imperial way.

On the other hand, every line of the Sphinx is curved and every surface is rounded and fluent. The entire statue indicates, nay, is eloquent with life, action, progress. In this connection the expression on the face of the sphinx is worthy of especial attention. It has a half-quizzical smile, subtle and many-sided. This smile speaks of a knowledge of the weaknesses of human nature, of the vicissitudes of man's life, yet is full of a sense of power and confidence in the outcome of the future. It is both realistic and optimistic.

As a whole, in time, as the traveler thus stands, observes, and meditates the huge body of the sphinx, rising out of the rock and sand of the desert, and looking out over the fertile plain of Egypt toward Jerusalem, becomes a symbol of the whole of human history.

As he stands there, observes, and ponders, the traveler recalls that each of the great historical world conquerors came at some

time during his career, stood before the sphinx and looked up into its face. Thus, thousands of years after the vast statue was made, Alexander the Great in the year 332 B.C. stood there and looked. By his position and attitude, if not in articulate words he asked the question "Am I the great hero, the ideal man you are looking for?" In silence the Sphinx continued to look toward Jerusalem.

Three hundred years later came Julius Caesar, stood in the same place, and, as it were, asked the same question with the same result.

Then over 1800 years afterward, came Napoleon Bonaparte and looked questioningly up into the face of the great image in the way shown in the painting by the artist Gerome; and still the wonderful stone countenance silently continued its gaze toward Jerusalem and the rising sun.

As he stands and observes perhaps one other idea comes to our traveler. The line along which the Sphinx is gazing if produced far enough, passes beyond Judea, across Persia, and the rest of Asia, across the Pacific Ocean to our own land, where first the social principles taught by Christ were embodied in a written political constitution, and where first the full fruits of these ideals should be realized on a large scale. In other words the sphinx has its back turned to imperialism, and is looking over and beyond the world conquerors, to the land of popular government.

Heliopolis and Karnak

Later our traveler goes to another place, visited by very few tourists and never even heard of by most of them, but which to the trained observer may well prove to be the most significant spot in Egypt. This is the site of the ancient city of Heliopolis. It lies a few miles to the North of Cairo, at the apex of the delta formed by the various streams into which the Nile separates before it enters the Mediterranean Sea. The site is to be carefully distinguished from the "New Heliopolis," a recently built suburb of Cairo.

The ruins left of the ancient city are few and scanty and the guidebook says but little about them and what they stand for. These surviving remains consist of a single obelisk and the remnants of some walls which rise but two or three feet above the level of the cultivated fields in which they lie.

So far as outward appearance is concerned it is hard to conceive of a greater contrast between these scanty ruins and the gigantic architectural remains at Karnak, which are to be seen at a point

400 miles from Cairo up the Nile. These are double starred by Baedeker as of unusual interest and merit, are advertised all over the world and are each year visited by multitudes of tourists. Here are to be seen the surviving remants of the most gigantic temples and their surroundings ever built by man. One propyleum or portal is 145 feet high and 375 feet wide. Here are to be seen avenues of columns and statues each 60 feet high and of equally massive sphinxes.

The most of this gigantic architecture was built by different conquering monarchs, like Rameses the Great, at a comparatively late stage in the history of the rising nation, the erections of one King often crowding upon and overlapping those of his predecessors. Across the river is the famous valley of the Kings, where their mummies were entombed with all the trappings of royalties, those found in the death chamber of King Tut being examples.

Stupendous and spectacular as are these ruins at Karnak and overwhelming to the senses, they are, after all, like the pyramids, a mere symbol of selfish imperialism and temporary material success. Most of the towering statues are those of the kings themselves, and on the walls are carved in pictures and hieroglyphs the boastful records of their royal achievements. Even the religion represented is largely an animalized one, mainly used as a means to the glorification of the monarch builders.

The despotism which produced these results made no worthwhile contribution to human welfare and progress, and like other despotisms soon passed into decay, leaving Egypt a weakened prey to every new invader.

Thus the remains at Karnak while profuse and mighty in outward appearance are scanty in inner ideal meaning and value. In contrast with them Heliopolis while meager in ruins is rich and mighty in ideal and spiritual meaning and value.

The word Heliopolis means "the city of the sun." This city arose in the earliest stages of Egyptian history and was the cultural capital of the nation for 2000 years. The three or four hundred feet of low masonry still found there was part of the foundations and walls of the world's first university, the first place where organized religion was developed, and where philosophy and science, including mathematics, were investigated and taught as abstract and systematized branches of knowledge. Thus Heliopolis was the birth place of formal education and real culture as part of the world's civilization.

This temple-university was already ancient, older in fact than Oxford is at the present time, when Moses, as a young prince of the house of Pharaoh, went there as a student to become "learned in all the wisdom of the Egyptians." Long before that, as the Bible tells us, Joseph had married a daughter of a priest-professor in this holy university town. Thither during many centuries came eager and gifted young men from other races and nations, to study and thence take to their home lands what they had learned. Among these were Pythagoras, Thales, Herodotus, and Plato.

As already stated the city continued in service as a center of religion of a high type and of education and general culture for a stretch of 2000 years. In the time of the Ptolemies it was gradually superseded by the neighboring university of Alexandria. In a sense indeed it is continued in the present university of Cairo. But in a larger sense through the culture and science which were diffused from it through the Greeks it became, by direct or indirect means, the parent source of all existing universities the world over. At least we may call these in some ways and to some degree its cultural offspring.

Obelisks

The intelligent and trained observer also finds the single obelisk which still stands at old Heliopolis quite as significant and full of meaning as the ruined walls found there. This obelisk was not erected by any egotistical monarch for his own glorification. The inscription on it dedicates it to the god Horus, the Rising Sun. In different parts of this inscription, Horus is termed or described as "The Sun of Life," "the one form (or principle) of life," "beloved, living forever, the Giver of Life."

In this connection it is worthy of note, that of primitive religions the one which forms the nearest approach to a realization of one universal divine spirit, or principle, taking many specific forms in different relations, and the kind constituting the most natural and easy route to such a lofty conception, is the worship of the sun. The traveler as he stands and looks at this obelisk in the Sun City, seems to see many of the more advanced thinkers that lived here, also standing here and reading this inscription and gradually rising to the pure and exalted conception of God described above. At any rate it is well known that many of the Egyptian priests in their private spiritual life, as kind of esoteric doctrine, had advanced to a form of pure monotheism. It is most suggestive to realize that

Moses was educated in such surroundings and to picture him as a young man standing before this monolith and reading and meditating upon this inscription. What were the thoughts, emotions, and ambitions, that stirred his soul as he thus stood and read?

Surely this one obelisk, as to outward form a slender and spiritualized pyramid, pointing like a finger heavenward, and by its inscription speaking an inspired and elevated message, has inexpressibly more of meaning and value than all of the colossal temples and sculptures at Karnak.

Continuing his investigations along this line, the traveler found that at various times, especially at the height of its greatness, many other obelisks were erected at Heliopolis besides the one which still stands there and which has just been described.

The largest obelisk in the world to-day is over 100 feet high and is now at Rome in front of the Lateran palace. Originally it was constructed by Thotmes III of Egypt about 1500 B.C. and stood in Heliopolis. It was brought to Rome by Constantine the Great. Two other of these pillars, also erected in this City of the Sun by this same Thotmes were later transported by the Roman emperor Augustus to Alexandria in 18 B.C. and put in a position to ornament and glorify the Caesareum, a temple which he had built there. Nearly 1900 years later, one of this pair of columns was transported to London where it now stands on the Thames embankment, and the other to New York and there placed in Central Park. Each of the two is now called by the name "Cleopatra's Needle."

These three obelisks thus going out at different times and in different ways, one to Rome, the capital city of the most powerful empire of ancient times and still in some senses a world capital; another to London, the center of the greatest empire of modern times, and the third to New York, the chief city of the world's greatest republic and leading commonwealth, seemed to the traveler to represent and be symbolic of the streams of culture and influence that have radiated from Egypt as the primal source of the world's civilization.

Heliopolis thus came to be realized by our visitor not only as the birthplace and home of the higher culture and civilization of Egypt but as the gateway through which these at various times have streamed out over the world.

The Soul of Civilization

In like manner the traveler found that in Palestine, in Greece, Constantinople, Italy, in all places, the categories and spirit of mathematics could be an organizing, idealizing, even spiritualizing influence or agency.

In recent years it has frequently been remarked that a certain lack of balance has arisen between the body and soul of the world's civilization. On the concrete and material side our world life has been expanding and developing in huge ways and at a tremendous speed, while at the same time there has been little change or advance in its ideal and spiritual elements. In other words, the greatest and most comprehensive need of the world at the present time is the development or even creation of an adequate soul for the great, new body of modern material civilization. Perhaps the mathematical categories and concepts, rightly utilized and applied, may have a worthwhile part in the process of solving this problem and meeting this need.

Viewed from this standpoint, the present two chief movements in mathematical progress which were mentioned at the outset, should in no way be antagonistic or mutually exclusive, but should stimulate, aid, and reinforce each other in important respects.

Errata

My attention has been called to some discrepancies in my article that appeared on pages 28 to 31 of the MATHEMATICS TEACHER for January, 1931, wherein I failed to eliminate the two variable parameters β and γ. After eliminating these and transforming the equations to a simple form (i.e., setting the origin at point B) we have for the locus of the bisectors of the angles

$$2xy(a-x) = h_a(ax - x^2 - y^2)$$

which is the equation of a cubic,

and for the locus of the orthocenters: $x^2 + h_a y = \frac{a^2}{4}$

which is the equation of a parabola.

A. BAKST

Galileo

It is frequently noted that Galileo Galilei was born on the day of Michelangelo's death (February 15, 1564), and that he died in the year of Newton's birth (1642). These facts are significant as emphasizing Galileo's position between the Renaissance with its reverence for the opinions of the Greek philosophers and the age of exact science with its demand for experimental confirmation of its theories. It has been said of Galileo that "He belongs among that small group of men such as Petrarch, Erasmus, and Voltaire, whose lives stand for a transitional epoch in the history of men's minds, who at once bear the marks of the age they help to supplant, and in their thoughts create and anticipate the development of succeeding ages."[1]

Galileo is associated with his native city Pisa where he discovered the laws of the pendulum and of falling bodies; with Padua where he taught from 1592-1610; and with Florence where he spent the latter part of his life. His tomb in the church of Santa Croce is close to that of his fourteenth century ancestor, also named Galileo, who is said to have excelled in mathematics and science, and who discarded his former surname for that of Galilei on being appointed to an important office in the city government.

Galileo's father had hoped that his son would become a merchant and so rebuild the family fortune, but seeing his son's great ability, he allowed him to study medicine and sent him to the University of Pisa. Galileo, however, became interested in mathematics and at the age of twenty-five, he accepted a poorly paid position as lecturer in mathematics at Pisa. Some years before (1581) he had begun his scientific discoveries by his observations of the vibration of a hanging lamp in the cathedral. Timing this by his pulse, he came to the conclusion that pendulums of equal length have equal periods of vibration. This led to further experimentation and to the formulation of the laws of the pendulum. The first observation, however, contradicted Aristotle's dictum that the heavier of two bodies would fall with the greater velocity. This contradiction brought Galileo to the study of falling bodies. In the fashion of his predecessors, he first formulated speculative arguments showing that Aristotle's reason-

[1] Martha Ornstein, *The Rôle of Scientific Societies in the Seventeenth Century*, Chicago, 1928, p. 23.

ing was untenable, and then in a way to be followed by later scientists, he tested his new hypotheses by actual experiments repeated many times, and leading to the adopting of new laws.

Galileo's career in Pisa ended when he tactlessly condemned an invention of a relative of the Medici family who had given him his place in the university. He then went to Padua where he invented sector compasses,[2] where he made the first thermometer, and where, hearing rumors of the invention of a telescope, he utilized his knowledge of lenses in making one for himself (1609). Galileo had previously taught the Ptolemaic theory of the planets feeling as he wrote Kepler (1597) that "his function as a professor demanded of him nothing but to hand down the accepted opinions of the past."[3] He seems to have been convinced himself of the truth of the Copernican theory. With the telescope, however, Galileo discovered the satellites of Jupiter which provided an apt illustration of the Copernican theory in the rotation of these "Medician stars" about the planet. Galileo was now invited to come to Florence by Cosimo de' Medici who had recently become Grand Duke. Cosimo II was ambitious to have his court excel in science as well as in art. This patronage gave Galileo a measure of protection without which much of his later work would have been impossible. He was now teaching the Copernican theory which had come under the censure of the Church, and in 1615 he was warned that he should limit himself to scientific research and keep from arguments that had a theological bias. The following year, he went to Rome to explain his views to the papal court.

In 1630, he completed his *Dialogo sopra dei due massini sistemi del mondo Tolemaico e Copernicano,* a conversation with three participants: a sponsor for the Ptolemaic theory, one for the Copernican theory, and an intelligent layman. The book was published in 1632 and the author was at once called to Rome and parts of the book were condemned as heretical. Galileo pleaded ill health but was obliged none the less to make the journey. He was tried for heresy and compelled to recant. It has been said that he immediately withdrew the recantation by whispering under his breath that the earth moved just the same, but this story is undoubtedly pure fiction.

Returning to Florence, Galileo settled again at Arcetri where he

[2] A translation of portions of Galileo's work on the proportional or sector compasses is given in *A Source Book in Mathematics,* edited by David Eugene Smith, New York, 1929, pp. 186-191.

[3] Ornstein, *loc. cit.* p. 218.

continued his studies in spite of increasing blindness which may have been due to the imperfections of his telescope. It was after he had lost his sight that he was visited by Milton who mentions this occasion in *Paradise Lost,* but Galileo's investigations continued with the help of Torricelli and Viviani and he studied the use of a pendulum to regulate a clock, and made various discoveries in mechanics and in hydrostatics.

A mathematical *Baedeker* would call one's attention to the hanging lamp in the cathedral in Pisa said by some to be the one which Galileo observed, but said by others to be of later date. It would note the Leaning Tower of Pisa as the site of the experiments with falling bodies. In Florence it would direct the visitor to the Torre di Gallo where Galileo made many observations in that city, and to the Tribuna di Galileo where are exhibited his telescope, his sector compasses, and other apparatus connected with Galileo or with his students. These aptly illustrate Professor Cajori's comment "Among his contemporaries it was chiefly the novelties he detected in the sky that made him celebrated, but J. Lagrance claims that his astronomical discoveries required only a telescope and perseverance, while it took extraordinary genius to discover laws from phenomena, which we see constantly and of which the true explanation escaped all earlier philosophers."[4]

[4] Florian Cajori, *A History of Mathematics,* New York, 1926, p. 172.

To Members of the National Council

It will help us greatly if members of the Council who are going to attend meetings of mathematics teachers during the remainder of the year will take it upon themselves to do a little advertising and publicity work. Subscription blanks describing the work of the Council will be sent to anyone upon request. We still hope to reach our goal of 10,000 members someday, but we cannot do it without your help.

THE EDITOR

NEWS NOTES

Sir Thomas Heath, who published his *History of Greek Mathematics* nine years ago, is now publishing with the Oxford University Press *A Manual of Greek Mathematics*. His idea in this book has been to give the general reader who has not lost interest in the studies of his youth and who would wish to know how it came about that a Greek by the name of Euclid wrote a textbook which, in an almost literal translation, was used in schools and in the universities in this country as the recognized basis of instruction in elementary geometry. He tells of Euclid's forerunners and of their respective contributions; and of Euclid's successor, Archimedes, who anticipated the integral calculus and who by arithmetic combined with geometry measured the circle and who laid the mathematical foundations of statics and created the whole science of hydrostatics.

OXFORD UNIVERSITY PRESS
114 Fifth Avenue, New York

Study of higher mathematics has suddenly grown so popular as to be "positively fashionable," declared Professor Edward V. Huntington of the department of mathematics at Harvard University in a radio talk last night (Wednesday, December 3) from station WEAF. Over 500 students at Harvard, mostly freshmen, elect a course in calculus every year, and probably half a million people throughout the country have studied that branch of mathematics, Professor Huntington said, adding that there were a half a million more who "appreciate how valuable a knowledge of the calculus would be if they had it."

Professor Huntington observed that a few decades ago only a few exceptional students "ever dared to rise to such dizzy heights" as the study of differential and integral calculus, but that today the calculus is merely the starting point in college mathematics. Students who specialize in mathematics are no longer regarded as hopelessly "queer," he said.

The change has come about, Professor Huntington asserted, through the discovery by a large and increasing number of people that higher mathematics is not only useful but beautiful.

Commenting on the newer uses of mathematics and its increasing usefulness, Professor Huntington referred particularly to its relation to long-distance communication. "In the field of long-distance telephony," he said, "mathematics has led the way to inventions which have greatly increased the distance the voice can be carried over a wire of given size. If it were not for mathematics you could not send a long-distance message today. The development of the radio itself has been directly dependent on mathematical-physical researches of the most advanced sort.

"Moreover, chemistry, biology, geology and even economics are coming more and more to realize that the rate of progress in these fields, under modern conditions, is directly proportional to the amount of mathematics they use."

Noting the value of the most recent mathematical theories on the analysis of statistical data to the world of business and finance, Professor Huntington remarked: "The mathematician of the future, instead of solving silly puzzles about 'how old is Ann,' may be using higher mathematical equations to figure out the next swing of the market.

"The first reason then for the changed attitude toward higher mathematics is the enormous extension of its usefulness in practical human affairs. The second reason for the new popularity of mathematics is more vital and powerful. Mathematics is not only useful; it is extremely beautiful. The beauty of a mathematical result is the fundamental motive for its pursuit. Every creative mathematician is essentially a creative artist."

Professor Huntington's talk was one of a series on science in its popular aspects by a number of leading scientists, broadcast every Wednesday evening under the auspices of the National Research Council Science Advisory Committee to the trustees of the 1933 Chicago World's Fair.

Eleventh Annual Ohio State Educational Conference

Speakers of both national and international note will address the Eleventh Annual Ohio State Educational Conference which will be held at Columbus, April 9, 10, 11, 1931. Charles Davila, Ambassador from Chile; Michael West, Principal of Teachers College, Dacca, India; and Newton D. Baker are among the more than one hundred speakers scheduled to participate in the program this year. An attendance of 6,000 is anticipated.

A section for guidance counselors will be added, making a total of thirty-six sectional divisions. These group meetings are provided to care for the varied interests of those in attendance. One or more meetings have been arranged for 35 sections. All meetings will focus upon the conference keynote for this year, *Individuality Through Guidance*.

◆ NEW BOOKS ◆

A Short History of Mathematics. By Vera Sanford. Houghton Mifflin Company, New York, 1930. xii + 402 pp. $3.25.

The appearance of a history of mathematics in one volume will be greeted with satisfaction in view of the increasing need for such a work. Perhaps no subject associated with elementary mathematics, using elementary as referring to mathematics through college grade, has increased in interest during the past twenty-five years as has this one. Great strides have been taken in placing this history on a foundation of scientific accuracy through a use of primary sources. The two-volume work of the distinguished historian, Professor David Eugene Smith, has set a standard which should be sought by other writers. The stamp of his authority has been put on the work under review through an introduction to it.

Many excellent features of Professor Smith's more elaborate treatise have been used in this one. We find a rich offering of maps, chronological details summarized in tables, facsimiles of early works, reproductions and pictures of mathematicians and of mathematical instruments. Some of these have not appeared before. The summaries at the end of each chapter commend themselves in particular. The characterization of the "Men Who Made Mathematics" may be cited as giving an especially good survey of the outstanding contributors to the development of the subject.

The topics covered may best be appreciated through a recital of the contents of the book: Men Who Made Mathematics; Arithmetic; Commercial Mathematics; Algebra; Verbal Problems; Practical Geometry; Demonstrative Geometry; Triogonometry; Analytic Geometry; Calculus; Theory of Numbers; Calculating Devices; Weights and Measures; The Place of Mathematics in the School Curriculum. General Histories of Mathematics and a Chronological Outline conclude the work. The extent of the treatment of each topic is better shown by the subtitles of the chapters of which one or two illustrations are given. Those under "Arithmetic" are: The Name Arithmetic; Number Magic; Number Systems; Computation with Ancient systems of Numerals; The Abacus; Hindu-Arabic Numerals; Computation with Hindu-Arabic Numerals; Fractions. The Chapter "Demonstrative Geometry" covers so wide a range of topics as: Early Greek Geometry; The Three Famous Problems; Euclid's "Elements"; Non-Euclidean Geometries; The Geometry of Conics; Modern Geometries. The good general survey under "Commercial Mathematics" is scarcely indicated even by the suggestive subtitles: General Survey; The Question of Interest; Investment Schemes of the Middle Ages; Modern Business.

The thorough acquaintance of the author with the field of the history of mathematics, and the consequent sense of authority which this work conveys, the freshness of treatment, the delightful style of the writing, all combine to make an unusually valuable and readable book. For the elementary and secondary school teacher, such a work as this affords a background of the subjects to be taught that is now considered essential by the leading authorities on requirements for teachers. There is little in the way of mathematical equipment for an understanding of what is here set forth that such a reader will not possess.

Even from the work of this author, there must be some dissenting opinions and those of the reviewer grow out of long experience in trying to make students in the classroom get the meaning out of such statements as will be cited. On p. 265 is found: "Hippocrates of Chios (*c.* 460 B.C.) had reduced the problem [Duplication of the Cube] of solving the equation $x^3 = 2a^3$ to finding two mean proportionals between the quantities a and $2a$." What Hippocrates did was to reduce the problem to one of plane geometry, namely, inserting two mean proportionals between the edge of the given cube and a line double that edge. The modern form may follow this if it is so stated explicitly. Again, on p. 257: " . . . . fifth century B.C. and from that time to the present the name 'Circle-squarers' has been applied to men who have attempted to do things that seem to be impossible." Has the term not rather been applied to men

who continue to assert that they can do these things after *science* has proved that they are impossible, making it an opprobrious term rather than a flattering one! On page 269: "In fact, it [Euclid's Elements] could aptly be called by the name 'General Mathematics.'" The publication of treatises covering different branches of mathematics under one cover does not fit into the blending of these branches, more or less successfully, which is understood by that designation. And what authority is there for the statements (p. 190) "This symbol *e* was chosen in honor of Euler . . . ." and (p. 19) "This author's [Al-Khowârizmî] great achievement was in writing a book (*c*.825) in which the Hindu numerals were used in the solution of equations" and others! There are few such instances as these but they go to show that care must be exercised in using any work. Statements must be verified by going to many sources.

A Short History of Mathematics appears in a series of books relating to mathematics put out in recent years under the editorship of John Wesley Young and published by Houghton Mifflin Company. It is an example of the fine book-making of this house. The editor and publisher are to be congratulated on his notable addition to the series.

LAO G. SIMONS
Hunter College
New York

OFFICIAL BALLOT

For the Election of Officers at the February 20, 1931, Meeting of the National Council of Teachers of Mathematics

For Second Vice-President, 1931-1933 (Vote for One)

☐ Hildebrandt, Martha
Maywood, Illinois

☐ Taylor, E. H.
Charleston, Illinois

For Members of the Board of Directors, 1931-1934 (Vote for Three)

☐ Gugle, Marie
Columbus, Ohio

☐ Sabin, Mary S.
Denver, Colorado

☐ Orleans, Joseph B.
New York, New York

☐ Thiele, C. Louis
Detroit, Michigan

☐ Poole, Hallie S.
Buffalo, New York

☐ Weimar, M. Bird
Wichita, Kansas

Please mark this ballot at once and mail same to Edwin W. Schreiber, Secretary, 314 N. Ward St., Macomb, Illinois.